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Experiments on convective layer formation and merging in a differentially heated slot

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This work describes experiments in which a stably stratified salt solution is subjected to steady lateral temperature gradients. The criteria for the onset of layered convection are observed to agree with previously published stability analysis. Convective layers formed in this way are found to be statically unstable; they always merge two into one to form a new system. This process continues until the density jump at the solute interface between layers is larger than the density deficit produced by the side wall heating/cooling. The eventual stable system develops into well-mixed convective layers separated by sharp solute interfaces. The conditions for any subsequent layer intermixing and its effect on the lateral heat transfer across the slot, are also described.

1. Introduction

Double-diffusive convection is a phenomenon that deals with the buoyancy-driven convection which results from the diffusion of two (or more) components with different molecular diffusivities in a fluid. Sometimes it is referred to as layered convection or cellular convection. It is also called thermohaline or thermosolutal convection if the relevant components are heat and salt (or solute).

If gradients in the component with larger species diffusivity are destabilizing, then two-dimensional convecting layers, separated by diffusion-limited interfaces, result. An example of this would be the heating of a stably stratified salt solution from below. Well-mixed convecting layers separated by thin, miscible interfaces result such that the density distribution becomes steplike.

Sufficiently strong lateral heating of a solution with uniform solute gradient also results in a layered convecting system. Here the resulting layer convection is a two-dimensional recirculating motion which is driven by the sideways heating. It may be shown, via stability analysis, that, for a given initially quiescent stratification, the layered system will form spontaneously when the sideways heating rate, as characterized by a thermal Rayleigh number, exceeds a critical value which is solely dependent on the initial stratification and fluid properties. The convecting layers which form are equal sized, and develop at a uniform rate.

Convective layer formation in a narrow slot subjected to steady, uniform heating

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was considered by Thorpe, Hutt & Soulsby (1969) and later by Hart (1971, 1973) using linear, and then finite, amplitude stability analysis coupled with experimental observations of heat/salt and heat/sugar systems. Their analytical and experimental results indicate that the layered system which forms at slightly supercritical conditions consists of a series of two-dimensional rolls with alternating sense of rotation as shown in Hart's figure 4 (1973). Hart's experimental observations indicated that the initially formed layers merged, two into one, to form a new system with twice the original wavelength. He conjectured that, at supercritical conditions, the cells with the correct sense of rotation would attain larger amplitudes than their neighbours, and entrain the latter. Subsequent layered systems were observed to consist of relatively well-mixed convecting regions separated by sharp solute interfaces. These convecting systems also merge two into one on a fairly regular time scale. His observations were that this occurred by penetration of vertically moving fluid adjacent to the side walls.

Wirtz & Liu (1975) have further studied the formation of convective layers in a narrow slot. In this case the time dependent formation of the layered system was simulated through numerical integration of the full set of governing differential equations. The critical thermal Rayleigh number dependence on initial stratification was determined to be in close agreement with the previous stability analysis.

At supercritical conditions the calculation produced layers with alternating sense of rotation. The kinetic energy of the initially formed layers steadily increased until the first two-into-one merging occurred. Subsequent, more precise, numerical calculations by Reddy (1978) have reproduced this earlier numerical result. A study of successive stream function isocontour maps led Reddy to conclude that the initial merging process occurs through a viscous entrainment process whereby convecting layers driven in the proper sense by the lateral heat transfer steadily erode the layers with opposite sense of rotation. This explanation was similar to the one offered earlier by Hart (1973).

Reddy's long-time calculations have shown that the first merging of layers results in well-mixed convective layers, which rotate in the correct sense, and which are separated by sharp steps in solute concentration. Any subsequent merging of adjacent convecting layers should then result from penetration of vertically moving fluid across this sharp interface. This is a different mechanism from the one that is responsible for the merging of the initially formed layered system. Therefore, one objective of the present work is to investigate the conditions and mechanisms for breakdown and merging of convecting layers produced and sustained by the lateral heating of a stably stratified fluid contained in a tall narrow slot.

Boyack & Kearney (1972) have shown that the lateral heat transfer across low aspect ratio enclosures ($Ar < 1$) containing single component fluids is strongly dependent on the enclosure aspect ratio. The kinematics of well-mixed convecting layers is very similar to the single component case. It is expected that the lateral heat transfer of a layered system would vary as the layer aspect ratio changed due to the merging process. This aspect of the heat transfer problem is also dealt with in this investigation.

In the next section we describe our apparatus and experimental procedure. This is followed with an analysis of results in which we show that the initially formed layers are always statically unstable; the merging process of the first type as described above must occur, and continue to occur until a statically stable layered system

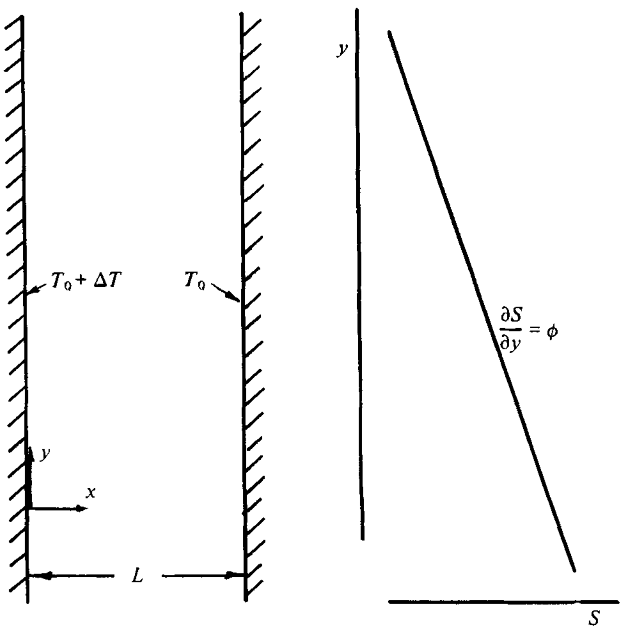

FIGURE 1. Idealized schematic diagram of a sideways heated slot containing a linearly stratified fluid.

results. At this point, the system will consist of a series of well-mixed layers separated by sharp interfaces. Subsequent intermixing occurs whenever the local stability parameter for a given interface approaches unity, resulting in penetration of fluid elements across the solute interface. We also show how the lateral heat transfer across the layered system varies. We find that each convecting layer behaves very much like an individual enclosure consisting of two isothermal side walls and an impermeable top and bottom. As the merging process proceeds the lateral heat transfer either increases or decreases, depending on whether the layer aspect ratio is less or greater than unity.

2. The physical experiments

An idealized diagram of the physical problem to be studied is shown in figure 1. It is a slot having two isothermal, very tall, vertical sidewalls maintained at temperatures T_0 and $T_0 + \Delta T$. The slot has width L and is filled with an aqueous solution of concentration S in such a way that S increases uniformly downward with constant gradient ϕ. At the onset of convective layer formation, the spatial distribution of T (temperature) and S within the slot is such that $\partial \rho / \partial x = 0$ with ρ the fluid density, given by

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]. \tag{1}$$

α and β are the coefficients of thermal and solute expansion and S_0, ρ_0 reference quantities.

The requirement for zero lateral density gradient gives the temperature and solute distribution within the fluid just before the onset of the instability as

$$T = T_0 + \frac{\Delta T}{L}\, x, \tag{2}$$

$$S = S_0 + \frac{\alpha \Delta T}{\beta L}\, x + \phi y. \tag{3}$$

The above conditions may be achieved throughout the bulk of the fluid by first filling the slot with a linearly stratified solute solution and then very slowly increasing the lateral temperature difference ΔT, over a protracted period of time such that quasi-steady conduction heat transfer and corresponding diffusion of S is maintained across the slot. The initial conditions embodied in (2) and (3) are not satisfied experimentally along the sidewalls since these walls are impermeable to S, with corresponding zero normal S derivative. This is apparently not of serious consequence since we were able to reproduce the neutral stability curve and initial layer wavelength as predicted analytically.

Consideration of the governing differential equations and their appropriate boundary conditions indicates that four independent parameters describe the problem. These are: a thermal Rayleigh number, $Ra = g\alpha\Delta T L^3/\nu D_T$, which is a measure of the destabilizing sideways temperature gradient, D_T being the thermal molecular diffusivity; a solute Rayleigh number, $Rs = g\beta\phi L^4/\nu D_S$, which is a measure of the stabilizing solute gradient, D_S is the solute molecular diffusivity; and Lewis and Prandtl numbers $Le = D_T/D_S$ and $Pr = \nu/D_T$ respectively. One dependent parameter of significance in this study is the slot Nusselt number, $Nu = FL/\Delta T k$, with F the average heat flux across the slot and k the fluid thermal conductivity (evaluated at mean temperature and solute concentration). Another is the dimensionless layer height or layer aspect ratio $Ar = h/L$.

The experiments which we performed to study the formation and subsequent evolution of sideways heated, convective layers utilized aqueous sugar solutions where $D_T \simeq 1{\cdot}4 \times 10^{-3}\,\text{cm}^2/\text{s}$ and $D_S \simeq 5 \times 10^{-6}\,\text{cm}^2/\text{s}$ (solute diffusivity), giving Lewis and Prandtl numbers of approximately 280 and 7 respectively.

The apparatus is shown in figure 2. It consists of two 40×15 cm composite copper plates separated by a gap which was varied from 2 to 4 cm. Thus our approximation of an infinitely tall slot was one with an overall aspect ratio which varied from 10 to 20. In an effort to avoid spurious results due to the top and bottom we always confined our observations to the middle two-thirds of the test section. Our experiments to determine the critical conditions for layer formation used only the larger aspect ratio configuration.

Each plate is backed by a well-baffled water duct used for circulation of heating/cooling water. Each composite plate consists of an inner 0·95 cm ($\frac{3}{8}$ in.) thick plate and an outer 0·61 cm ($\frac{1}{4}$ in.) plate sandwiching a 0·64 cm thick layer of cured epoxy which was cast in place. The thermal conductivity of the cured epoxy was determined so that the average heat flux through each sidewall may be determined by measuring the temperature difference between the inner and outer copper plates.

Five carefully grounded copper/constantan thermocouple junctions are embedded along the vertical centre-line in each of the four copper plates. These are wired in

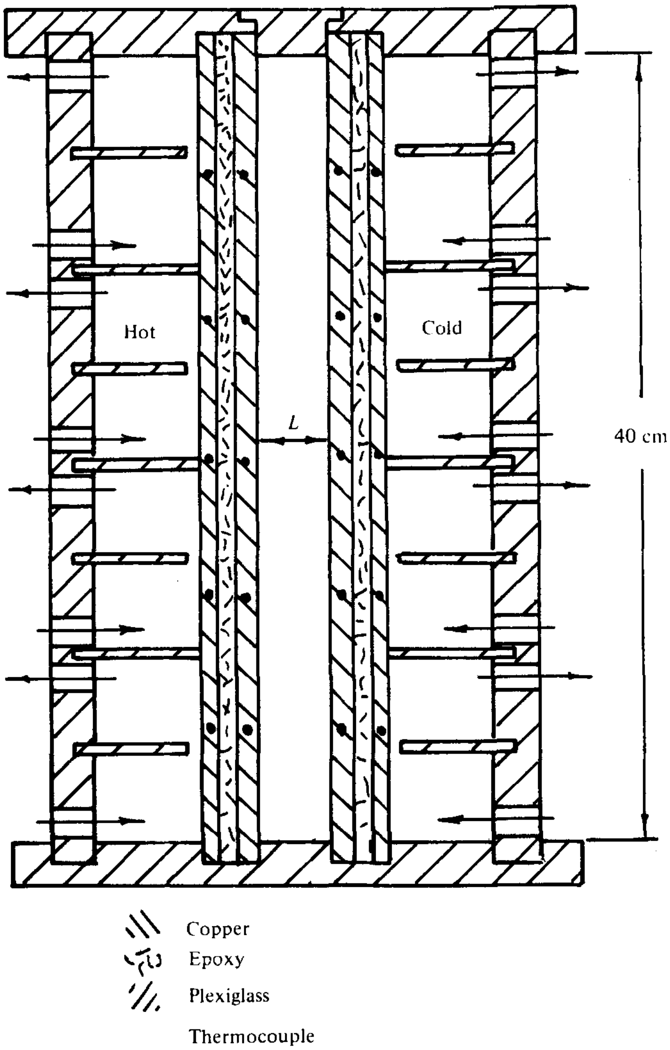

FIGURE 2. Sketch of a side view of the experimental apparatus. The slot width, L, was varied between 2 and 4 cm, resulting in an aspect ratio ranging from 10 to 20.

differential fashion to a 24 point recorder such that the recorder servo-switching mechanism monitors the drop through each plate, ΔT_w, and the drop across the slot, ΔT. In this way both heat flux and slot temperature difference are continuously monitored.

Each circulating passage for each copper wall is supplied by water from a large (15 gal.) reservoir. The cold reservoir temperature is kept constant by an immersion cooler operated in conjunction with a heater/circulator with temperature control by a proportional controller. The hot bath temperature and hence the temperature increase across the slot is maintained by a 1000 W heater immersed in the hot reservoir and controlled by another proportional controller. The input signal for this controller consists of five pairs of differential thermocouples which are located along the vertical centre-line of each of the inner ($\frac{3}{8}$ in.) copper plates. These thermocouple junction pairs

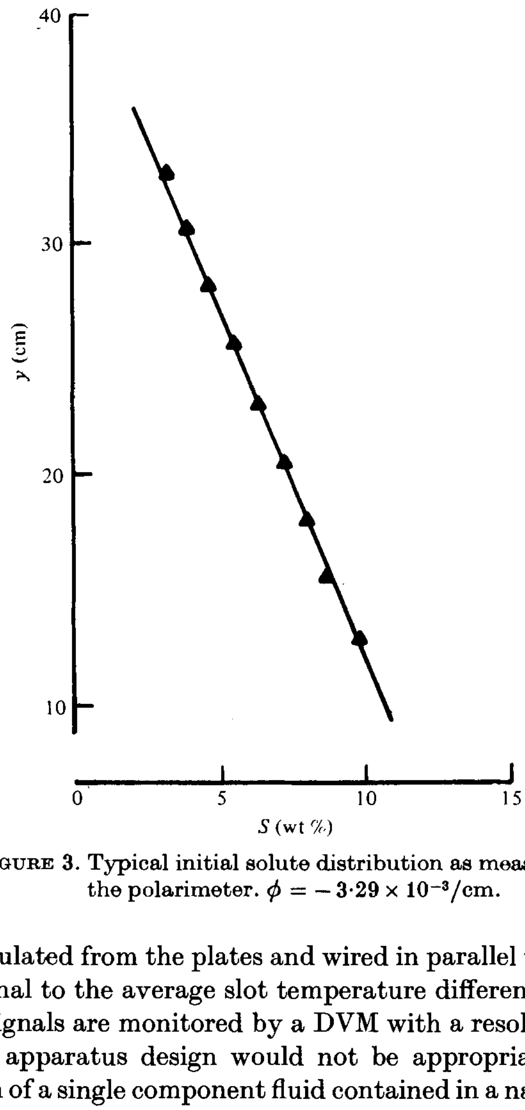

FIGURE 3. Typical initial solute distribution as measured by the polarimeter. $\phi = -3{\cdot}29 \times 10^{-3}$/cm.

are electrically insulated from the plates and wired in parallel to the controller, giving a signal proportional to the average slot temperature difference. For the purposes of data analysis all signals are monitored by a DVM with a resolution of $\pm 1\,\mu$V.

This particular apparatus design would not be appropriate for experiments on natural convection of a single component fluid contained in a narrow slot since it would not be possible to maintain the inner ($\frac{3}{8}$ in.) copper plates isothermal over their height. However, the layered system is such that a new convective system repeats itself every 5 mm to 5 cm. This periodicity is such that we were able to maintain the isothermal condition of each plate to within less than a measured value of $\pm 3{\cdot}5\,\%$ of the applied ΔT at incipient layer formation when lateral heat transfer rates are essentially by conduction. At later times, when the convecting system consists of a series of well-mixed layers separated by sharp interfaces, the heat load is greater (3–15 times the conduction value). Under these conditions the two inner plates were determined to be isothermal to within less than $\pm 10\,\%$ of the applied ΔT for each experiment.

Our measurement of average heat flux was based on five temperature drop measurements across the epoxy layer of each composite plate. These are designated as ΔT_w. When these ten signals are recorded on a strip chart, the resulting data has a scatter of approximately $\pm 15\,\%$ about a mean value. Therefore, we estimate that our indication of average wall heat flux, which is directly proportional to the average

ΔT_w, is to within $\pm 15\%$ when well-mixed layers exist. The average Nusselt number is essentially the ratio of the composite wall average temperature drop, ΔT_w, divided by the slot temperature difference, ΔT. The tolerance on the average Nusselt numbers reported here then becomes the r.m.s. of the two relative errors, or approximately $\pm 18\%$.

Aqueous sugar solutions are bifringent. The angle of polarization of polarized light will rotate as the light traverses a sugar solution, with the amount of rotation linearly proportional to the optical path length and the sugar concentration. This fact was used in the determination of the vertical solute distribution within the test section using a polarimeter of our own design (Wirtz & Reddy 1974). A linearly polarized beam from a He–Ne laser was projected through the test section and into a turning head which contained a Glan–Thompson prism in front of a photomultiplier tube. The turning head was capable of rotating the prism with a precision of 0·01°. The prism was rotated until the light intensity falling on the photomultiplier tube was minimum, in this way determining the polarization angle of the light leaving the test section. The change in polarization angle of the beam as it crossed the test section was used to determine the average sugar concentration existing along the beam. We estimate that our apparatus is capable of determining sugar concentration to within $\pm 0{\cdot}1\%$ wt with a spatial resolution of about 1 mm.

The test section was filled with a linear sugar stratification using the continuous filling technique. After a suitable waiting period (usually about 1 h) the solute distribution was determined as described above. A representative set of measurements is shown in figure 3 where the gradient was determined by a least squares fit of the data points.

3. Experimental results – initial layered system

We have performed a number of experiments on the initial formation and growth of the layered system with $L = 2$ cm. In these cases it is important to duplicate the initial conditions discussed previously by very gradually increasing the slot temperature difference up to the point of the onset of the instability. We set our temperature control system so that the slot would make a very gradual approach to the critical state. This is shown in figure 4, which is a tracing of the locus of data points from a representative strip chart recording for part of one experiment. In actuality our recorder plots a colour-coded dot for each datum point every 3 s. The two bands indicated in figure 4 are hand-traced envelopes of these points as a function of time. The band labelled ΔT represents the five independent measurements of the slot temperature difference. The band labelled ΔT_w represents the ten temperature drops measured across the epoxy layer of the two composite walls. The indicated ΔT_w is therefore directly proportional to the average wall heat flux, F, while the ratio of the two curves is proportional to the average Nusselt number. The data shown is for a slot width, $L = 2$ cm, resulting in a characteristic thermal diffusion time of approximately 0·8 h. Each major division in the vertical represents 1 h while each major division in the horizontal represents 0·05 mV (or approximately 1·2 °C). Throughout the experiments periodic observations (about every $\frac{1}{2}$ h) are made of the test section using a shadowgraph; otherwise everything is covered with a minimum of 25 mm of styrofoam insulation. The primary visual observation was the layer count in the middle two-

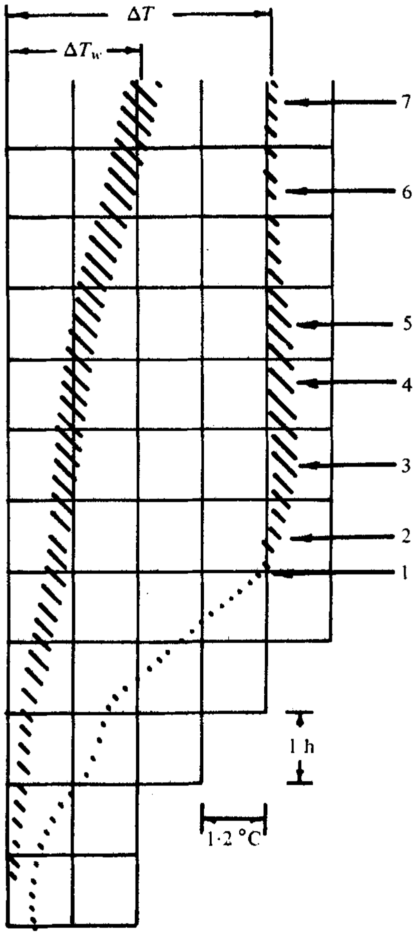

FIGURE 4. Tracing of a typical strip chart recording of slot temperature difference ΔT and average wall heat flux (proportional to ΔT_w). Expt. no. 7, $-Rs = 6{\cdot}27 \times 10^8$.

Point no.	$Ra \times 10^{-5}$	Nu	Average Ar
1	5·65†	1·31	—
2	5·86	1·06	0·17†
3	6·14	1·31	0·18
4	6·10	1·57	0·33
5	5·99	1·82	0·38
6	5·68	2·75	0·49
7	5·76	3·24	0·60

† Denotes experimentally observed critical state.

TABLE 1. Experimental results for expt. no 7 shown in figure 4. $Rs = -6{\cdot}27 \times 10^8$.

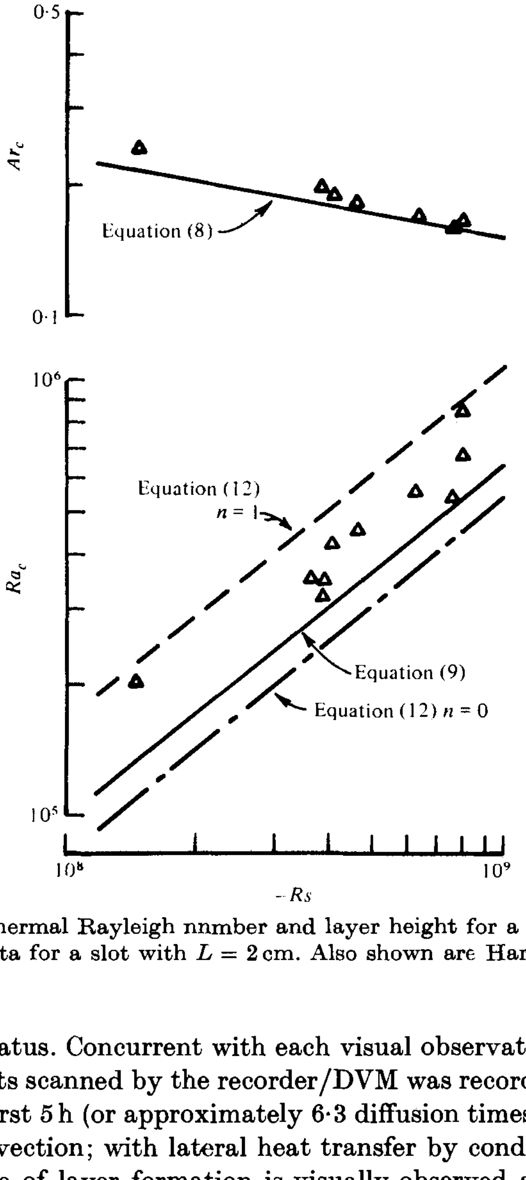

FIGURE 5. Critical thermal Rayleigh nnmber and layer height for a sideways heated slot. △ represents present data for a slot with $L = 2$ cm. Also shown are Hart's (1973) analytical predictions (——).

thirds of the apparatus. Concurrent with each visual observation, the mV output of each of the 24 points scanned by the recorder/DVM was recorded.

In figure 4, the first 5 h (or approximately 6·3 diffusion times) are characterized by the absence of convection; with lateral heat transfer by conduction across the slot. At point 1 evidence of layer formation is visually observed as a spatially periodic tonal variation on the shadowgraph. This is designated as the critical condition. Within 30 min (point 2 in figure 4) the tonal variation has intensified sufficiently so that the critical layer size may be determined. This is indicated in table 1 which is a tabulation of results obtained from figure 4 and our visual observations made over the central two-thirds of the apparatus where Ar is the average layer height scaled by the slot width L. Properties have been evaluated at mean solute concentration and temperature of the apparatus. Shadowgraphs of these initially formed layers have appeared previously in the literature (see, for example, Thorpe *et al.* 1969).

The experimentally determined critical Rayleigh number Ra_c for initial layer formation of dimensionless height Ar_c, over a decade variation in $-Rs$, are shown in figure 5. The experimental values are slightly higher than those predicted by Hart's

(1973) asymptotic theory [(8) and (9)]. This is probably due to the practical difficulty involved in correctly judging the occurrence of the critical state in that we probably exceed it before we are aware it has occurred.

Our apparatus had no provision for directly determining the sense of rotation of these initially formed layers; however, Hart's (1973) apparatus did. The agreement between our measured values of Ar_c with his, together with numerical results discussed in the introduction, lead us to believe that these initially formed layers exhibited alternating sense of rotation. During this phase the convection is apparently very weak. Numerical calculations by Reddy (1978) indicate maximum convective velocities of order 0·02 cm/s when $L = 2$ cm. Our shadowgraph observations indicate that these initially formed convective layers do not develop very sharp interfaces. Instead the odd numbered cells grow at the expense of the even numbered ones so that, after 2·75 h (point 4 in figure 4), the layered system consists of half the number of layers, with average height equal to twice the critical value.

An explanation of the cause of the first merging process may be obtained by considering the static stability of two adjacent convecting layers each of height h. If we imagine that the fluid within each newly formed convecting layer is allowed to begin to mix as the layer forms, then the vertical density distribution would tend toward a staircase shape with the change in density across the interface separating layers given by

$$\Delta\rho_1 = -\beta\phi h. \tag{4}$$

On the other hand, the destabilizing change in density due to the sideways heating is given by

$$\Delta\rho_2 = \alpha\Delta T. \tag{5}$$

We form the ratio

$$R_\rho = \frac{\Delta\rho_1}{\Delta\rho_2} = \frac{-\beta\phi h}{\alpha\Delta T}, \tag{6}$$

called a stability number, which is a simple measure of the static stability of a fluid parcel. Recent numerical studies of convection in side-heated layered systems containing steplike solute distributions (Wirtz 1977), together with our experimental results to be presented in the next section, have shown that if $R_\rho \leqslant 1$ the fluids will mix.

Substituting our parameters Rs, Ra, etc., gives the requirement that

$$R_\rho = \frac{(-Rs)\,Ar}{Ra\,Le} > 1, \tag{7}$$

for static stability. For the initially formed layers, Hart (1973) has shown that

$$\frac{2\pi}{Ar_c} = (-\tfrac{1}{2}\pi^2\,Rs)^{\frac{1}{6}}, \tag{8}$$

and

$$Ra_c = \frac{(2\pi^4)^{\frac{1}{6}}\,6^{\frac{1}{2}}(-Rs)^{\frac{5}{6}}}{Le-1}, \tag{9}$$

for the dimensionless initial layer height and critical thermal Rayleigh number. Substituting (8) and (9) into requirement (7) indicates that at critical conditions

$$R_{\rho_c} = \frac{2}{6^{\frac{1}{2}}}\,\frac{Le-1}{Le} < 1, \tag{10}$$

which is valid for all $Le > 1$. Equation (10) indicates that convective layers formed by the steady application of a destabilizing temperature gradient across a slot containing a uniformly stratified fluid will always be statically unstable, and thus always must merge as soon as fluid within each convecting layer begins to mix.

This result may be restated by identifying Ra_n as the thermal Rayleigh number required to give convective layers with dimensionless heights which are 2^n multiples of the critical dimensionless layer size, Ar_c, given by (8):

$$Ar_n = 2^n Ar_c, \quad n = 0, 1, \ldots . \tag{11}$$

Substitution of (11) and (8) into (7) gives

$$Ra_n < \frac{2^{n+\frac{7}{6}} \pi^{\frac{2}{3}}}{Le} (-Rs)^{\frac{5}{6}}. \tag{12}$$

The limiting condition for inequality (12) is plotted in figure 5. Note that $Ra_0 < Ra_c$ for all $(-Rs)$ so that if $Ra \geqslant Ra_c$ the layers which form will immediately merge to form a new system with approximately twice the original dimensionless layer height. If $Ra > Ra_1$ then this new system will again immediately merge to form layers with four times the original layer size.

If however $Ra_c < Ra < Ra_1$, as is the case for the data of figure 4 and table 1, the newly formed layers with $Ar \cong 2Ar_c$ will continue to develop with a consequent sharpening of the interfaces separating convecting layers. This is observed as a sharpening of the tonal variation in light intensity of our shadowgraph image. For the data depicted in figure 4 and table 1 this occurred at a time labelled point 4 with the average layer aspect ratio, $Ar = 0{\cdot}33$. All subsequent visual observations indicated the presence of sharp interfaces separating convecting layers. Figure 4 and table 1 indicate that the average aspect ratio doubles again within the next 4 h (point 7). However, this figure represents convective layers ranging in size up to 1·15. The rate of intermixing of layers separated by sharp interfaces is not uniform over the height of the apparatus and is due to the local value of R_ρ for two adjacent convecting layers becoming less than unity. This occurs in a finite height apparatus due to vertical transport of solute. Layer merging of this sort propagates in from the top and bottom of the apparatus as the available supply of solute at apparatus bottom is exhausted and as solute collects at the top of the apparatus.

4. Experimental results – layers with sharp interfaces

In order to study the conditions for intermixing of well mixed convective layers within the slot, we artificially establish a steplike solute distribution which would be heated at constant $Ra > Ra_c$. The initial conditions were such that at the onset $R_\rho > 1$, however, due to the vertical transport of solute across the interface, R_ρ approached unity. In every case investigated over a layer aspect ratio range

$$2{\cdot}2 < Ar < 6{\cdot}7,$$

the interfaces mixed at $R_\rho \simeq 1$. The details of the mixing process are shown in figure 6 (plate 1) for $Ra = (2{\cdot}4 \pm 0{\cdot}2) \times 10^7$.

In this case the slot width, L, is 4 cm and the initial $R_\rho = 1{\cdot}025$ across the interface by direct measurement at the start of the experiment.

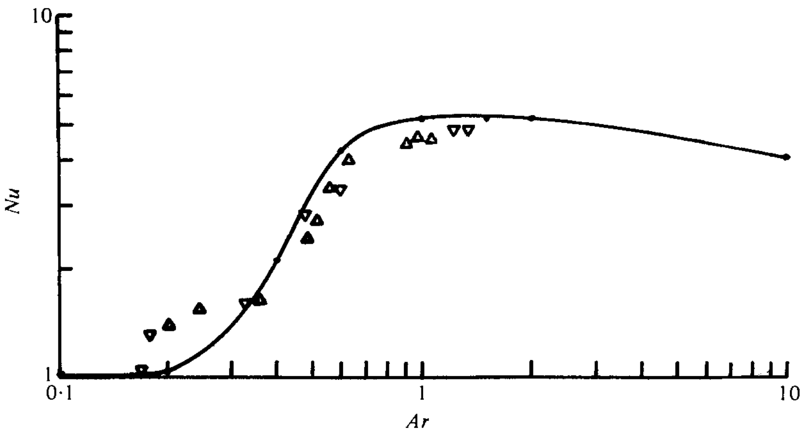

FIGURE 7. Average wall Nusselt number dependence on layer aspect ratio at slightly supercritical conditions. △ = Expt. no. 4, $-Rs = 3{\cdot}86 \times 10^8$, $Ra = (3{\cdot}6 \pm 0{\cdot}4) \times 10^5$, ▽ = Expt. no. 7, $-Rs = 6{\cdot}27 \times 10^8$, $Ra = (5{\cdot}9 \pm 0{\cdot}25) \times 10^5$, and —— a numerical calculation by Reddy for well-mixed layers separated by sharp interfaces with $Ra = 2 \times 10^5$, $R_\rho = 2$.

Figure 6(*a*) shows a stable interface about 5·30 h after the experiment had started. At 6·23 h [figure 6(*b*)] $R_\rho \simeq 1$, the interface is quasi-stable at this point. It experiences large amplitude undulations. However fluid from one layer is not observed to penetrate through the interface to the other layer. Twenty-two minutes later [figure 6(*c*)] penetration is seen to occur adjacent to the isothermal side boundaries. In this way fluid steadily bleeds into the adjacent layer. It is carried vertically along the isothermal side boundary and mixes with the fluid of lower solute concentration in the central quiescent core of each convecting layer. In the case shown, the penetrating up-flow is apparently greater than penetrating down-flow. As a result the remaining interface shifts downward as shown in figure 6(*d*) at 6·67 h. This occurs because the property ratio β/α decreases with increasing temperature. Consequently the local value of R_ρ for the interface is always minimum near the hot wall. Since β/α also decreases with increasing S, this effect also results in overturning of interfaces near the bottom of the apparatus first. The whole process is relatively rapid so that by 6·87 h the two layers have completely mixed as shown in figure 6(*e*).

Each time two convective layers intermix, the lateral heat transfer across the slot is affected. This may be seen in figure 4 as a gradual increase in the average wall heat flux curve with time. This increase in average lateral heat transfer with layer merging is shown in figure 7 where the measured average Nusselt number is cross plotted versus the observed average layer aspect ratio for two different sets of data. Also shown is Reddy's (unpublished) numerical calculation for a system consisting of well mixed layers separated by sharp interfaces. In the figure we see that the lateral heat transfer rapidly increases as the layer aspect ratio increases up to $Ar \simeq 1$. This behaviour is very similar to that observed for natural convection of single-component fluids in rigid containers consisting of two isothermal sidewalls and adiabatic top and bottom (Boyack & Kearney 1972). Since at this point the convective layers are well mixed, the presence of solute does not affect the wall Nusselt number and the only apparent difference between the present case and the single-component case is that the

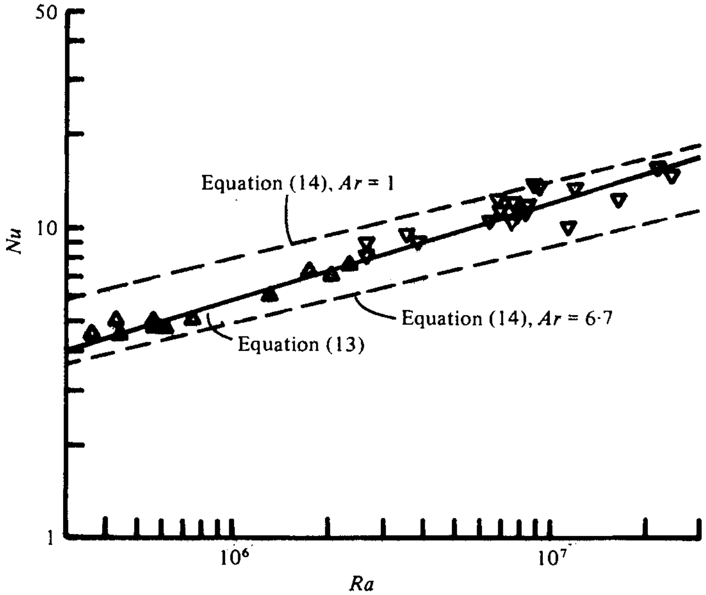

FIGURE 8. Average wall Nusselt number for layered convection across a slot with $1 < Ar < 6{\cdot}7$. $\triangle$: layers formed from continuous gradient, ∇ : layers established during filling.

former consists of a flexible, conducting top and bottom boundary. Successive intermixing of convective layers with $Ar > 1$ should lead to a decrease in Nu as is observed for the single-component case and the numerical calculation shown in figure 7 although our data for $1 < Ar < 6{\cdot}7$ is sufficiently scattered that no clear trend may be discerned.

The fact that the layered system behaves very much like a number of single-component enclosures acting in parallel is also shown in figure 8 where we have plotted the average slot Nusselt number versus Ra for all of our data with $1 < Ar < 6{\cdot}7$. A least squares fit of these points gives

$$Nu = 0{\cdot}082 Ra^{0{\cdot}31}. \tag{13}$$

This compares favourably with previously published correlations obtained for a single-component fluid. For example for $Ar > 1$ and moderate to high Prandtl numbers McGregor & Emery (1968) proposed

$$Nu = 0{\cdot}25 Ra^{0{\cdot}25} Ar^{-0{\cdot}25}, \quad 10^4 < Ra, \tag{14}$$

appropriate to laminar flow. This correlation evaluated at $Ar = 1$ and $Ar = 6{\cdot}7$ is plotted for comparison in figure 8.

5. Conclusions

Experimentally determined values of critical thermal Rayleigh number Ra_c and dimensionless cell height Ar_c for the onset of layered convection of a stratified fluid in a narrow slot subject to a steadily applied temperature gradient are in good agreement with stability theory. The differences between theory and experiment are probably due to experimental difficulties in assessing the critical state. Our experimental procedure did not allow for direct measurement of the convective velocity in the initially formed layers. However, on the basis of the agreement of our measurements with previous theoretical and experimental (Hart 1973) and numerical (Wirtz & Liu 1975; Reddy 1978) results, we presume that the initial layered system consists of very weak convection with alternating sense of rotation.

Convective layers formed via this mechanism are always statically unstable. Hence the newly formed convective layers do not develop into well-mixed cells separated by sharp interfaces. Instead they immediately merge, two into one, to form a new layered system. Depending on the magnitude of the applied thermal Rayleigh number this process repeats itself until a statically stable layered system is formed. This occurs when the density jump due to the solute step between layers is greater than the density deficit produced by the sideways heating. These static conditions are embodied in (12).

Statically stable layered systems eventually develop such that well-mixed cells separated by sharp density interfaces result. Any subsequent layer intermixing occurs when the local stability number for a given interface drops to unity. Mixing is then by penetration of vertically moving fluid through the interface adjacent to the side boundaries of the slot. In a finite height enclosure all layers must eventually intermix since vertical transport will eventually use up the available supply of solute at the bottom of the apparatus. We conjecture that in an infinite height system, or one with appropriate source and sink for solute, a stable layered system would be achieved.

Well-mixed convective layers behave very much like solid boundary enclosures containing single-component fluids. The lateral heat transfer across a slot containing a layered system with $Ar < 1$ rapidly increases as adjacent convective layers intermix. Based on heat transfer measurements of single-component fluids in high aspect ratio enclosures it is expected that the average heat transfer of a layered system with large aspect ratio would decrease due to layer intermixing. For the layer aspect ratio range investigated in this study this effect is weak.

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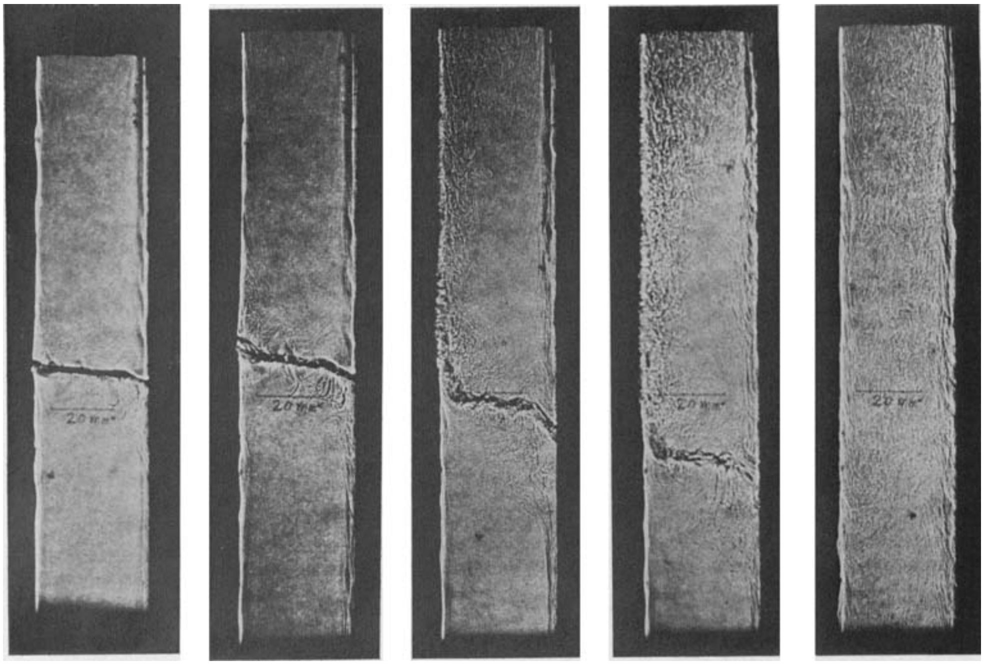

(*a*) (*b*) (*c*) (*d*) (*e*)

FIGURE 6. Shadowgraph views of interfacial mixing between two well-mixed convective layers. $Ra = (2 \cdot 4 \pm 0 \cdot 2) \times 10^7$.